

# **NCATE/NCTM Program Standards (2003) Programs for Initial Preparation of Mathematics Teachers**

## *Standards for Middle Level Mathematics Teachers*

### **Process Standards (Standards 1-7)**

The process standards are based on the belief that mathematics must be approached as a unified whole. Its concepts, procedures, and intellectual processes are so interrelated that, in a significant sense, its “whole is greater than the sum of the parts.” This approach would best be addressed by involvement of the mathematics content, mathematics education, education, and field experience faculty working together in developing the candidates’ experiences.

Likewise, the response to the disposition standard will require total faculty input. This standard addresses the candidates’ nature and temperament relative to being a mathematician, an instructor, a facilitator of learning, a planner of lessons, a member of a professional community, and a communicator with learners and their families.

#### **Standard 1: Knowledge of Mathematical Problem Solving**

Candidates know, understand, and apply the process of mathematical problem solving.

##### **Indicators**

- 1.1 Apply and adapt a variety of appropriate strategies to solve problems.
- 1.2 Solve problems that arise in mathematics and those involving mathematics in other contexts.
- 1.3 Build new mathematical knowledge through problem solving.
- 1.4 Monitor and reflect on the process of mathematical problem solving.

#### **Standard 2: Knowledge of Reasoning and Proof**

Candidates reason, construct, and evaluate mathematical arguments and develop an appreciation for mathematical rigor and inquiry.

##### **Indicators**

- 2.1 Recognize reasoning and proof as fundamental aspects of mathematics.
- 2.2 Make and investigate mathematical conjectures.
- 2.3 Develop and evaluate mathematical arguments and proofs.
- 2.4 Select and use various types of reasoning and methods of proof.

**Standard 3: Knowledge of Mathematical Communication**

Candidates communicate their mathematical thinking orally and in writing to peers, faculty, and others.

**Indicators**

- 3.1 Communicate their mathematical thinking coherently and clearly to peers, faculty, and others.
- 3.2 Use the language of mathematics to express ideas precisely.
- 3.3 Organize mathematical thinking through communication.
- 3.4 Analyze and evaluate the mathematical thinking and strategies of others.

**Standard 4: Knowledge of Mathematical Connections**

Candidates recognize, use, and make connections between and among mathematical ideas and in contexts outside mathematics to build mathematical understanding.

**Indicators**

- 4.1 Recognize and use connections among mathematical ideas.
- 4.2 Recognize and apply mathematics in contexts outside of mathematics.
- 4.3 Demonstrate how mathematical ideas interconnect and build on one another to produce a coherent whole.

**Standard 5: Knowledge of Mathematical Representation**

Candidates use varied representations of mathematical ideas to support and deepen students' mathematical understanding.

**Indicators**

- 5.1 Use representations to model and interpret physical, social, and mathematical phenomena.
- 5.2 Create and use representations to organize, record, and communicate mathematical ideas.
- 5.3 Select, apply, and translate among mathematical representations to solve problems.

**Standard 6: Knowledge of Technology**

Candidates embrace technology as an essential tool for teaching and learning mathematics.

**Indicator**

- 6.1 Use knowledge of mathematics to select and use appropriate technological tools, such as but not limited to, spreadsheets, dynamic graphing tools, computer algebra systems, dynamic statistical packages, graphing calculators, data-collection devices, and presentation software.

**Standard 7: Dispositions**

Candidates support a positive disposition toward mathematical processes and mathematical learning.

**Indicators**

- 7.1 Attention to equity
- 7.2 Use of stimulating curricula
- 7.3 Effective teaching
- 7.4 Commitment to learning with understanding
- 7.5 Use of various assessments
- 7.6 Use of various teaching tools including technology

**Pedagogy (Standard 8)**

In addition to knowing students as learners, mathematics teacher candidates should develop knowledge of and ability to use and evaluate instructional strategies and classroom organizational models, ways to represent mathematical concepts and procedures, instructional materials and resources, ways to promote discourse, and means of assessing student understanding. This section on pedagogy is to address this knowledge and skill.

**Standard 8: Knowledge of Mathematics Pedagogy**

Candidates possess a deep understanding of how students learn mathematics and of the pedagogical knowledge specific to mathematics teaching and learning.

**Indicators**

- 8.1 Selects, uses, and determines suitability of the wide variety of available mathematics curricula and teaching materials for all students including those with special needs such as the gifted, challenged and speakers of other languages.
- 8.2 Selects and uses appropriate concrete materials for learning mathematics.
- 8.3 Uses multiple strategies, including listening to and understanding the ways students think about mathematics, to assess students' mathematical knowledge.
- 8.4 Plans lessons, units and courses that address appropriate learning goals, including those that address local, state, and national mathematics standards and legislative mandates.
- 8.5 Participates in professional mathematics organizations and uses their print and on-line resources.
- 8.6 Demonstrates knowledge of research results in the teaching and learning of mathematics.
- 8.7 Uses knowledge of different types of instructional strategies in planning mathematics lessons.
- 8.8 Demonstrates the ability to lead classes in mathematical problem solving and in developing in-depth conceptual understanding, and to help students develop and test generalizations.

8.9 Develop lessons that use technology's potential for building understanding of mathematical concepts and developing important mathematical ideas.

## **Content (Standards 9-15)**

Candidates' comfort with, and confidence in, their knowledge of mathematics affects both what they teach and how they teach it. Knowing mathematics includes understanding specific concepts and procedures as well as the process of doing mathematics. That knowledge is the subject of the following standards.

### **Standard 9: Knowledge of Number and Operation**

Candidates demonstrate computational proficiency, including a conceptual understanding of numbers, ways of representing number, relationships among number and number systems, and meanings of operations.

#### **Indicators**

- 9.1 Develop the mathematics that underlies the procedures used for operations involving whole numbers, integers, and rational numbers.
- 9.2 Use properties involving number and operations, mental computation, and computational estimation.
- 9.3 Provide equivalent representations of fractions, decimals, and percents.
- 9.4 Create, solve, and apply proportions.
- 9.5 Apply the fundamental ideas of number theory.
- 9.6 Make sense of large and small numbers and use scientific notation.
- 9.7 Analyze and explain the distinctions among whole numbers, integers, rational numbers, and real numbers and whether or not the field axioms hold.
- 9.8 Demonstrate knowledge of the historical development of number and number systems including contributions from diverse cultures.

### **Standard 10: Knowledge of Different Perspectives on Algebra**

Candidates emphasize relationships among quantities including functions, ways of representing mathematical relationships, and the analysis of change.

#### **Indicators**

- 10.1 Explore, analyze, and represent patterns, relations, and functions.
- 10.2 Represent and analyze mathematical structures.
- 10.3 Investigate equality, equations, and proportional relationships.
- 10.4 Use mathematical models to represent quantitative relationships.
- 10.5 Analyze change in various contexts.
- 10.6 Demonstrate knowledge of the historical development of algebra including contributions from diverse cultures.

**Standard 11: Knowledge of Geometries**

Candidates use spatial visualization and geometric modeling to explore and analyze geometric shapes, structures, and their properties.

**Indicators**

11.1 Demonstrate knowledge of core concepts and principles of Euclidean geometry in two and three dimensions.

11.2 Exhibit knowledge of informal proof.

11.3 Build and manipulate representations of two- and three-dimensional objects and perceive an object from different perspectives.

11.4 Specify locations and describe spatial relationships using coordinate geometry.

11.5 Analyze properties and relationships of geometric shapes and structures.

11.6 Apply transformation and use congruence, similarity, and line or rotational symmetry.

11.7 Demonstrate knowledge of the historical development of Euclidean and non-Euclidean geometries including contributions from diverse cultures.

**Standard 12: Knowledge of Calculus**

Candidates demonstrate a conceptual understanding of limit, continuity, differentiation, and integration and a thorough background in the techniques and application of the calculus.

**Indicators**

12.1 Demonstrate a conceptual understanding of basic calculus concepts.

12.2 Demonstrate knowledge of the historical development of calculus including contributions from diverse cultures.

**Standard 13: Knowledge of Discrete Mathematics**

Candidates apply the fundamental ideas of discrete mathematics in the formulation and solution of problems.

**Indicators**

13.1 Demonstrate a conceptual understanding of the fundamental ideas of discrete mathematics such as finite graphs, trees and combinatorics.

13.2 Use technological tools to apply the fundamental concepts of discrete mathematics.

13.3 Demonstrate knowledge of the historical development of discrete mathematics including contributions from diverse cultures.

**Standard 14: Knowledge of Data Analysis, Statistics, and Probability**

Candidates demonstrate an understanding of concepts and practices related to data analysis, statistics, and probability.

- 14.1 Design investigations, collect data through random sampling or random assignment to treatments, and use a variety of ways to display the data and interpret data representations.
- 14.2 Draw conclusions involving uncertainty by using hands-on and computer-based simulation for estimating probabilities and gathering data to make inferences and decisions.
- 14.3 Identify misuses of statistics and invalid conclusions from probability.
- 14.4 Use appropriate statistical methods and technological tools to analyze data and describe shape, spread, and center.
- 14.5 Investigate, interpret, and construct representations for conditional probability, geometric probability, and for bivariate data.
- 14.6 Demonstrate knowledge of the historical development of probability and statistics including contributions from diverse cultures.

### **Standard 15: Knowledge of Measurement**

Candidates apply and use measurement concepts and tools.

#### **Indicators**

- 15.1 Recognize measurement attributes and their effect on the choice of appropriate tools and units.
- 15.2 Apply techniques, tools, and formulas to determine measurements.
- 15.3 Employ estimation as a way of understanding measurement units and processes.
- 15.4 Complete error analysis through determining the reliability of the numbers obtained from measurement.
- 15.5 Demonstrate knowledge of the historical development of measurement and measurement systems including contributions from diverse cultures.

### **Field-Based Experiences (Standard 16)**

The development of mathematics teacher candidates should include opportunities to examine the nature of mathematics, how it should be taught and how students learn mathematics; observe and analyze a range of approaches to mathematics teaching and learning, focusing on the tasks, discourse, environment and assessment; and work with a diverse range of students individually, in small groups, and in large class settings.

### **Standard 16: Field-Based Experiences**

Candidates complete field-based experiences in mathematics classrooms.

#### **Indicators**

- 16.1 Engage in a sequence of planned opportunities prior to student teaching that includes observing and participating in middle grades mathematics classrooms under the supervision of experienced and highly qualified teachers.
- 16.2 Experience full-time student teaching in middle grades mathematics that is supervised by an experienced and highly qualified teacher and a university or college supervisor with middle grades mathematics teaching experience.

16.3 Demonstrate the ability to increase students' knowledge of mathematics.